THE TRUE DURATION OF THE IMPULSE RESPONSE USED TO ESTIMATE REVERBERATION TIME

Laurent Faiget, Robert Ruiz and Claude Legros
L.A.U.T.M., Université de Toulouse-le Mirail
5, Allées A. Machado, 31058 Toulouse Cedex France
faiget@cict.fr

ABSTRACT

Since Schroeder [1], reverberation time can be computed from the impulse response. Some other studies have shown the advantages of this method leading often to more accurate results than the classical ones. But, it can be shown that, when background noise is an important energetic part of the impulse response, the Schroeder's method must be revised. A theoretical approach is proposed showing the influence of background noise and the integration boundaries in the Schroeder's equation. Hence, a new upper limit of integration is defined based on regression lines to model background noise and decay. The intersection defines t' being the useful length of the impulse response. Experiments were processed in a large factory hall. Comparisons between classical reverberation time measurements and Schroeder's method are made showing good results even for the lowest values of signal-to-noise ratio.

1. INTRODUCTION

Since many years, some works have shown the importance of the impulse response in room acoustics. Many criteria describing the quality of a room are derived from the squared impulse response. Schroeder [1] has proposed a new method for measuring reverberation time based on the backward integrated impulse response. This method is often used giving satisfying results. But some problems exist when the measure is corrupted by background noise [2], [3], [4]. Different techniques deal with this problem by substraction or truncation of background noise [2]. But errors in reverberation time calculation still remain.

In this paper, we present a theoretical approach of the error made on the reverberation time calculation using the backward integrated impulse response leading to a temporal limit defined as the useful length of the impulse response.

2. THEORETICAL APPROACH

An ideal squared impulse response can be simply written in an exponential form when the sound field is supposed to be diffuse. It can be modelled as follows :

$$h^2(t) = E_0 \cdot e^{-kt} \tag{1}$$

where E_0 is the energy at $t=0$ and $\frac{k}{2}$ the damping constant.

2.1. Pure exponential decay

Calculation of reverberation time using Schroeder's equation [1] yields to :

$$\langle s^2(t) \rangle = N \int_t^{\infty} h^2(\tau)\, d\tau \tag{2}$$

and

$$\langle s^2(t) \rangle = N \cdot \frac{E_0}{k} \cdot e^{-kt} \tag{3}$$

and

$$L(t) = 10 \log \left(N \cdot \frac{E_0}{k} \right) + 10 \log \left(e^{-kt} \right) \tag{4}$$

In practical cases, the upper limit of integration of eq. (2) must be revised because a real measurement reaches only a finite value T, duration of acquisition. Eq. (2) becomes :

$$\langle s_T^2(t) \rangle = N \int_t^{T} h^2(\tau)\, d\tau \tag{5}$$

$$\langle s_T^2(t) \rangle = N \cdot \frac{E_0}{k} \cdot \left(e^{-kt} - e^{-kT} \right) \tag{6}$$

Taking ten times the logarithm of eq. (6) :

$$L_T(t) = 10 \log(N \cdot \frac{E_0}{k}) + 10 \log(e^{-kt} - e^{-kT}) \tag{7}$$

Eq. (7) shows that for large values of T, $L_T(t) \simeq L(t)$ and therefore error on reverberation time calculation will tend to zero.

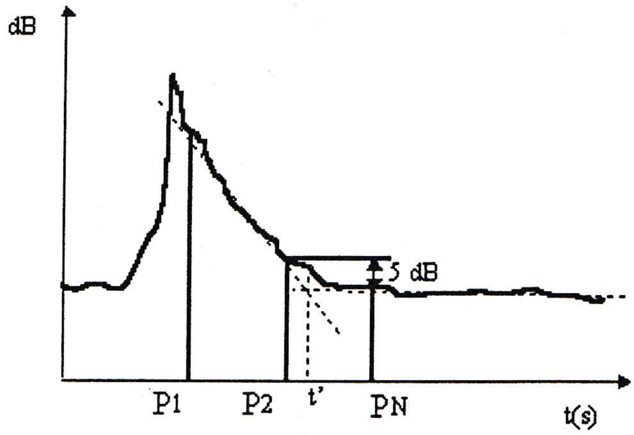

Figure 1. Determination of t' on the logarithmic plot of the squared impulse response

2.2. Exponential decay and background noise

Adding noise to the pure exponential decay yields to a new formulation of the squared impulse response :

$$h_N^2(t) = E_N + E_0 \cdot e^{-kt} \tag{8}$$

where E_N is an additional noise energy.

The backward integrated impulse response taken over a finite time of acquisition T becomes :

$$\langle s_{N,T}^2(t) \rangle = N \int_t^T h_N^2(\tau)\, d\tau \tag{9}$$

$$\langle s_{N,T}^2(t) \rangle = N \cdot E_N (T - t) + N \cdot \frac{E_0}{k} \cdot (e^{-kt} - e^{-kT}) \tag{10}$$

So, the level becomes:

$$\begin{aligned} L_{N,T}(t) &= 10 \log(N \cdot E_N(T-t) \\ &\quad + N \cdot \frac{E_0}{k}(e^{-kt} - e^{-kT})) \end{aligned} \tag{11}$$

Examining eq. (11), one can see that the term $E_N(T-t)$, which is a noise term, affects the form of the decay curve. So, the upper limit of integration T of eq. (9), is to be revised.

3. USEFUL LENGTH OF THE DECAY CURVE

On the logarithmic plot of the squared impulse response $h_N^2(t)$, a new temporal limit t' is defined leading to a noise truncation (Figure 1). Regression lines are made on both decay and background noise. The beginning point of the decay regression (P_1) is based on the mean free path. To avoid the first singular reflections, this point is chosen just after the direct sound plus six times the time corresponding to mean free path [5]. The end point (P_2) is chosen 5 dB above the background noise. Regression line of the background noise starts at point (P_N), chosen manually where the decay seems to reach the noise, ending at the end of the acquisition. The intersection of the two regression lines gives the t' value.

4. ERROR ON REVERBERATION TIME

The impulse response is now shortened at t'. So, eq. (9) becomes :

$$\langle s_{N,t'}^2(t) \rangle = N \int_t^{t'} h_N^2(\tau)\, d\tau \tag{12}$$

$$\langle s_{N,t'}^2(t) \rangle = N \cdot E_N(t' - t) + N \cdot \frac{E_0}{k} \cdot (e^{-kt} - e^{-kt'}) \tag{13}$$

Taking ten times the logarithm of eq. (13) :

$$\begin{aligned} L_{N,t'}(t) &= 10 \log(N \cdot E_N(t'-t) \\ &\quad + N \cdot \frac{E_0}{k}(e^{-kt} - e^{-kt'})) \end{aligned} \tag{14}$$

Errors expressed in dB can be written from eq. (4), eq. (11) and eq. (14) :

$$\varepsilon_T(t) = L_{N,T}(t) - L(t) \tag{15}$$

$$\begin{aligned} \varepsilon_T(t) &= 10 \log(N \cdot E_N(T-t) \\ &\quad + N \cdot \frac{E_0}{k} \cdot (e^{-kt} - e^{-kT})) \\ &\quad - 10 \log(N \cdot \frac{E_0}{k} e^{-kt}) \end{aligned} \tag{16}$$

and

$$\varepsilon_{t'}(t) = L_{N,t'}(t) - L(t) \tag{17}$$

$$\begin{aligned} \varepsilon_{t'}(t) &= 10 \log(N \cdot E_N(t'-t) \\ &\quad + N \cdot \frac{E_0}{k} \cdot (e^{-kt} - e^{-kt'})) \\ &\quad - 10 \log(N \cdot \frac{E_0}{k} e^{-kt}) \end{aligned} \tag{18}$$

Considering eq. (16) and eq. (18), it can be seen that the term $N \cdot E_N(T - t)$ will always be greater than the term $N \cdot E_N(t' - t)$ for a given time t. So the relative error $\varepsilon_{t'}$, which is the difference between $L_{N,t'}(t)$ and the theoretical curve $L(t)$, will be always less than $\varepsilon_T(t)$ over time range 0 to t'. But this error cannot be estimated precisely.

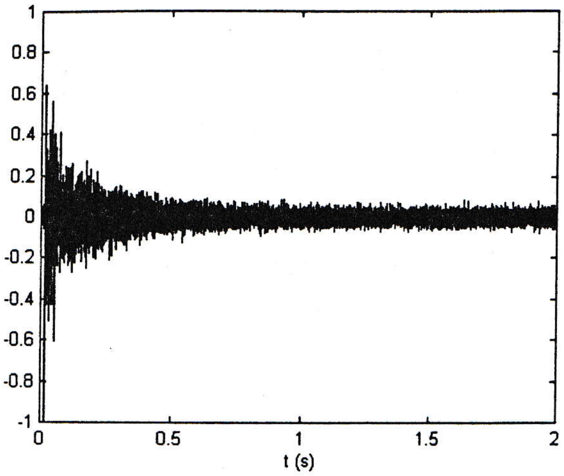

Figure 2. Example of a measured impulse response corrupted by noise

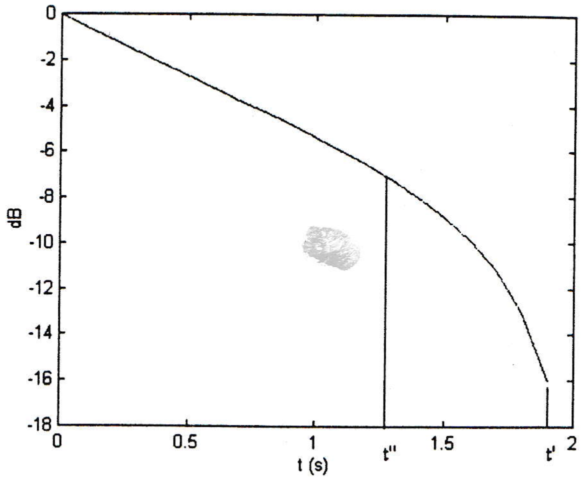

Figure 3. Logarithmic plot of the Schroeder's equation showing t" and t'

It can be seen also that $L_{N,t'}(t)$ decreases steadily until a value closed to t' and then decreases to infinity at t'. So, $L_{N,t'}(t)$ can be modelled as a straight line until a value called t'' just less than t' (see Figure 3). A slope, p, is computed from $L_{N,t'}(t)$:

$$p = \frac{1}{t''} \cdot 10 \log \left(\frac{E_N(t' - t'') + \frac{E_0}{k} \cdot (e^{-kt''} - e^{-kt'})}{E_N \cdot t' + \frac{E_0}{k} \cdot (1 - e^{-kt'})} \right) \tag{19}$$

$e^{-kt'}$ is a negligible value as well as $(e^{-kt''} - e^{-kt'})$ because in large reverberant rooms $kt' \gg 1$ and $Kt'' \gg 1$. Eq. (19) can be rewritten :

$$p \simeq \frac{1}{t''} \cdot 10 \log \left(\frac{E_N(t' - t'')}{E_N \cdot t' + \frac{E_0}{k}} \right) \tag{20}$$

So, $L_{N,t'}(t)$ takes a new formulation which is an approximation :

$$\hat{L}_{N,t'}(t) = \frac{1}{t''} \cdot 10 \log \left(\frac{E_N(t' - t'')}{E_N \cdot t' + \frac{E_0}{k}} \right) \cdot t + 10 \log \left(N \cdot E_N \cdot t' + N \cdot \frac{E_0}{k} \right) \tag{21}$$

5. RESULTS

Measurements were made in a large factory hall (2000 m^3). Ten linear reverberation times were measured in two different points using white noise excitation. These values will be taken as reference values for comparisons. Furthermore, at the same points, impulse responses were measured with an MLS equipment leading to a 2 s signal acquisition at sample frequency 16 kHz. In this experiments, three different signal-to-noise ratios were obtained, one for point 1 and two for point 2. Point 2 was measured with and without background noise by adding white noise to the sequence. An example of an impulse response is given in Figure 2. The signal-to-noise ratios (column 1 of Table 1) are the difference between the relative levels of points P_1 and P_2 (see Figure 1). The values of t' are obtained from Section 3. RT_{60} (column 3 of Table 1) are computed from the reverse-time integration of the squared impulse response with a upper limit of integration equal to t'. Column 4 is the average of ten reverberation time analogical measurements with their standard deviation in column 5.

	S/B (dB)	t' (s)	RT_{60}	$< RT_{60} >$	σ (%)
1	12.5	0.53	1.90	1.93	8.8
2	18.3	0.78	2.06	1.90	8.9
3	11.0	0.53	1.92	1.90	8.9

Table 1. Results for three different measurements in a large factory hall.

Figure 4 shows an example of the determination of the reverberation time from the logarithmic plot of the squared impulse response for point 1. The Schroeder's equation with the upper integration limit equal to the total duration ($T = 2s$) gives the upper curve in Figure 4 leading to an overestimated reverberation time. On the other hand, with the upper limit of integration equal to t' (lower curve in Figure 4), the computed reverberation time is very close to the analogical one (see Table 1).

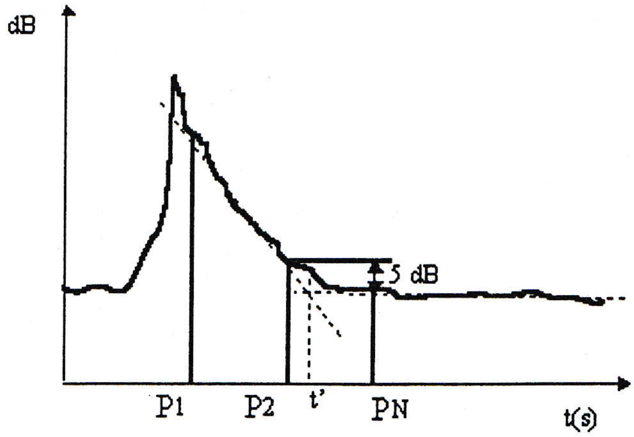

Figure 1. Dtermination of t' on the logarithmic plot of the squared impulse response

2.2. Exponential decay and background noise

Adding noise to the pure exponential decay yields to a new formulation of the squared impulse response :

$$h_N^2(t) = E_N + E_0 \cdot e^{-kt} \tag{8}$$

where E_N is an additional noise energy.

The backward integrated impulse response taken over a finite time of acquisition T becomes :

$$\langle s_{N,T}^2(t) \rangle = N \int_t^T h_N^2(\tau)\, d\tau \tag{9}$$

$$\langle s_{N,T}^2(t) \rangle = N \cdot E_N(T-t) + N \cdot \frac{E_0}{k} \cdot (e^{-kt} - e^{-kT}) \tag{10}$$

So, the level becomes:

$$\begin{aligned} L_{N,T}(t) &= 10\log(N \cdot E_N(T-t) \\ &\quad + N \cdot \frac{E_0}{k}(e^{-kt} - e^{-kT})) \end{aligned} \tag{11}$$

Examining eq. (11), one can see that the term $E_N(T-t)$, which is a noise term, affects the form of the decay curve. So, the upper limit of integration T of eq. (9), is to be revised.

3. USEFUL LENGTH OF THE DECAY CURVE

On the logarithmic plot of the squared impulse response $h_N^2(t)$, a new temporal limit t' is defined leading to a noise truncation (Figure 1). Regression lines are made on both decay and background noise. The beginning point of the decay regression (P_1) is based on the mean free path. To avoid the first singular reflections, this point is chosen just after the direct sound plus six times the time corresponding to mean free path [5].

The end point (P_2) is chosen 5 dB above the background noise. Regression line of the background noise starts at point (P_N), chosen manually where the decay seems to reach the noise, ending at the end of the acquisition. The intersection of the two regression lines gives the t' value.

4. ERROR ON REVERBERATION TIME

The impulse response is now shortened at t'. So, eq. (9) becomes :

$$\langle s_{N,t'}^2(t) \rangle = N \int_t^{t'} h_N^2(\tau)\, d\tau \tag{12}$$

$$\langle s_{N,t'}^2(t) \rangle = N \cdot E_N(t'-t) + N \cdot \frac{E_0}{k} \cdot (e^{-kt} - e^{-kt'}) \tag{13}$$

Taking ten times the logarithm of eq. (13) :

$$\begin{aligned} L_{N,t'}(t) &= 10\log(N \cdot E_N(t'-t) \\ &\quad + N \cdot \frac{E_0}{k}(e^{-kt} - e^{-kt'})) \end{aligned} \tag{14}$$

Errors expressed in dB can be written from eq. (4), eq. (11) and eq. (14) :

$$\varepsilon_T(t) = L_{N,T}(t) - L(t) \tag{15}$$

$$\begin{aligned} \varepsilon_T(t) &= 10\log(N \cdot E_N(T-t) \\ &\quad + N \cdot \frac{E_0}{k} \cdot (e^{-kt} - e^{-kT})) \\ &\quad - 10\log(N \cdot \frac{E_0}{k} e^{-kt}) \end{aligned} \tag{16}$$

and

$$\varepsilon_{t'}(t) = L_{N,t'}(t) - L(t) \tag{17}$$

$$\begin{aligned} \varepsilon_{t'}(t) &= 10\log(N \cdot E_N(t'-t) \\ &\quad + N \cdot \frac{E_0}{k} \cdot (e^{-kt} - e^{-kt'})) \\ &\quad - 10\log(N \cdot \frac{E_0}{k} e^{-kt}) \end{aligned} \tag{18}$$

Considering eq. (16) and eq. (18), it can be seen that the term $N \cdot E_N(T-t)$ will always be greater than the term $N \cdot E_N(t'-t)$ for a given time t. So the relative error $\varepsilon_{t'}$, which is the difference between $L_{N,t'}(t)$ and the theoretical curve $L(t)$, will be always less than $\varepsilon_T(t)$ over time range 0 to t'. But this error cannot be estimated precisely.

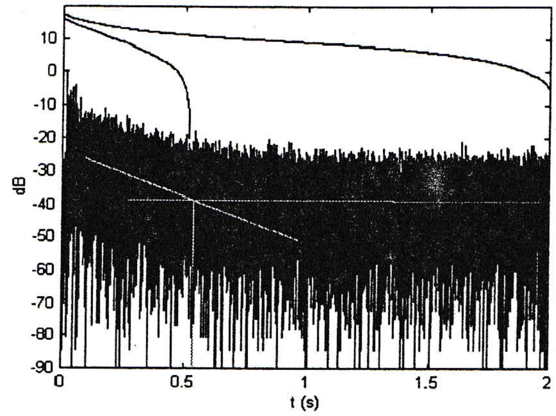

Figure 4. Determination of the reverberation time from the logarithmic plot of the squared impulse response

6. CONCLUSION

The theoretical approach shows that upper limit of integration and background noise are two important parameters in the reverberation time computation. The first conclusion is that the impulse response must be noisyless to obtain good results. Hence, when the background noise is not important, the upper limit of integation is not a real problem (see eq. (7)). The signal acquisition must be large enough not to truncate the decay. But in real situation, in factory hall for example, high level background noise made by machines is always present and often, it is impossible to stop it. The study presented here shows the importance of this extraneous noise on the reverberation time computation (see eq. (16) and eq. (18)). The limit t' defined in this paper represents a noise truncation but not a noise substraction which allows to model the Schroeder's equation as a straight line. Results presented in Table 1 show that even for low signal-to-noise ratios, the computed reverberation times are in the 95% confidence intervals of the analogical ones. Some other experiments are in progress to validate this method for larger reverberation times and lower signal-to-noise ratios. Furthermore, this method can also be used to compute energy ratios taken from the squared impulse response leading to acoustical criteria [6], avoiding errors due to the late noisy part.

REFERENCES

[1] M. R. Schroeder, "New method for measuring reverberation time," J. Acoust. Soc. Am., vol. 37, pp. 409-412, 1965.

[2] W. T. Chu, "Comparison of reverberation measurements using Schroeder's impulse method and decay-curve averaging method," J. Acoust. Soc. Am., vol. 63 (5), pp. 1444-1450, 1978.

[3] M. Vorländer and H. Bietz, "Comparison of methods for measuring reverberation time," Acustica, vol. 80, pp. 205-215, 1994.

[4] N. Xiang, "Evaluation of reverberation times using a nonlinear regression approach," J. Acoust. Soc. Am., vol. 98 (4), pp. 2112-2121, 1995.

[5] H. Kuttruff, "A simple iteration schemes for the computation of decay constants in enclosures with diffusely,reflecting boundaries," J. Acoust. Soc. Am., vol 98 (1), pp. 288-293, 1995.

[6] L. Faiget, R. Ruiz and C. Legros, "Estimation of impulse response length to compute room acoustical criteria," Forum Acusticum, Antwerpen, Belgium, April 1996.